**MATH2019 (Term 1, 2019) – Writing Assignment**

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**Question (1), Part (a):**

Question:

Given is a symmetric matrix *A*. The matrix *A* is:

*A* has three distinct eigenvalues; two of which are known:

and an unknown eigenvalue *k*. This question requires the calculation of *k*.

Answer:

There exists a diagonal matrix *D* where its diagonal entries consists of the eigenvalues of *A*. The matrix *D* is:

The trace (sum of main diagonal entries) of *A* is equal to the trace of *D*. Therefore *k* can then be calculated from the following:

Therefore the value of the third eigenvalue *k*, for the matrix *A*, is -196.

**Question (1), Part (b):**

Question:

Following the question above, it is given that *A* also has three known linearly independent eigenvectors:

with corresponding eigenvalues:

This question requires the unknown orthogonal matrix *Q* for the diagonal matrix *D*, such that:

.

Answer:

The set of eigenvectors of *A*, , are orthogonal because *A* is a symmetric matrix and has three distinct eigenvalues. But is not orthonormal therefore an orthogonal matrix *Q* cannot be obtained since an orthogonal matrix is a square matrix that has columns and rows which are orthonormal vectors.

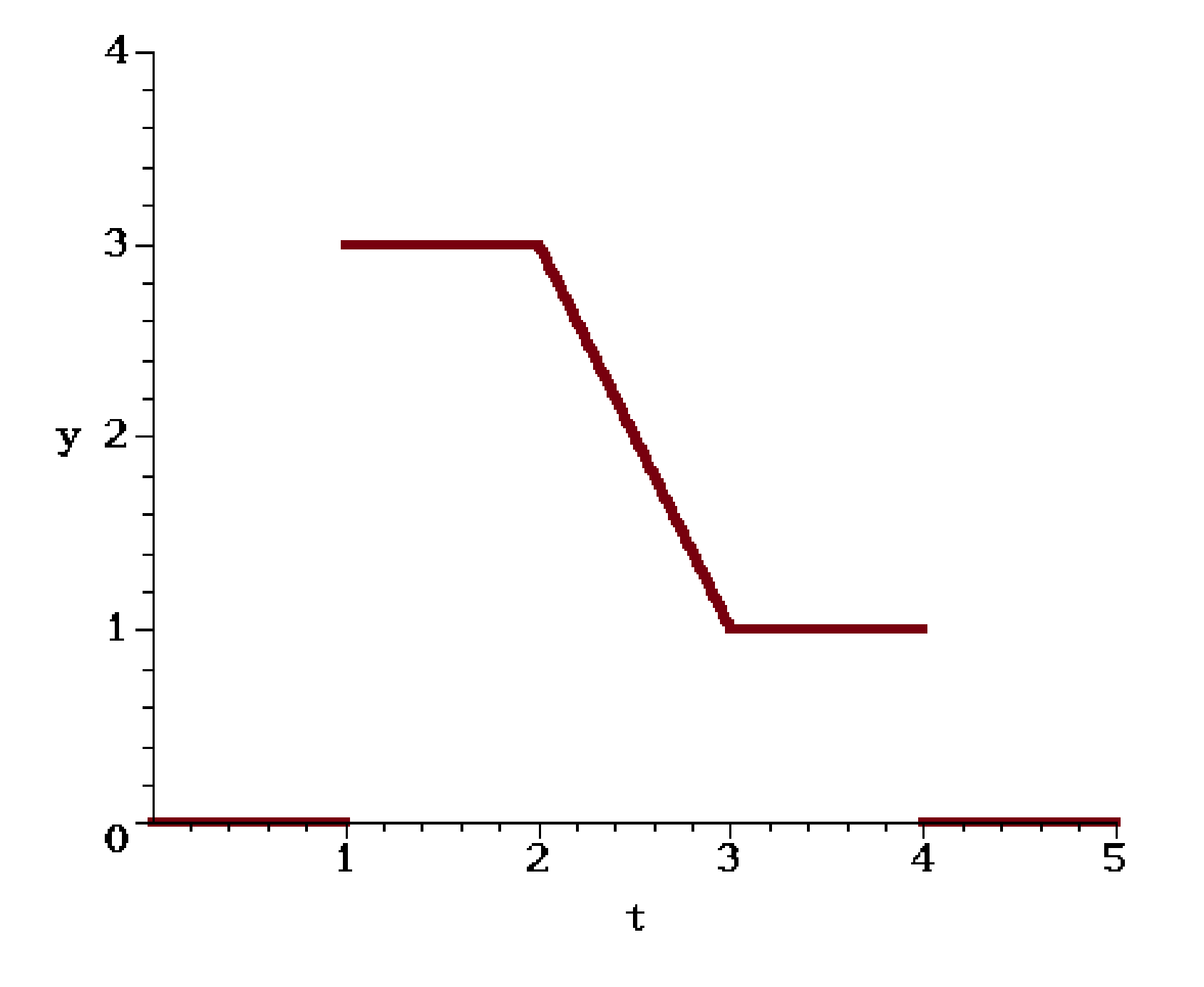
To determine the unit eigenvectors; the magnitude of each eigenvector is:

A vector divided by its magnitude gives a unit vector which has a magnitude of one. Therefore the orthonormal set of eigenvectors are:

The orthonormal set of eigenvectors of *A* with respect to its corresponding eigenvalues in *D* make up the columns of the orthogonal matrix *Q*:

**Question (2):**

Question:



**Figure 1 – graph of y(t)**

This question requires the function of the above graph (figure 1) to be expressed using Heaviside function notation.

The Heaviside function is:

where *c* is a real constant.

Answer:

By considering each piece of the graph in figure 1 to get an equation for a particular domain:

For

For :

For

The line is calculated using the two-point formula:

and picking two points lying on the line:

So,

For :

For :

can now be expressed as a piece-wise function:

Consider the piece-wise function and when each piece of “starts” and “stops”. For example:

The Heaviside expression for an interval with unit value is:

where *a* and *b* are real constants.

The interval has an equivalent Heaviside expression of:

Over this interval, the function is:

Therefore the Heaviside expression for is:

That is, the function is which starts at and stops at .

Expressing as a Heaviside expression:

is the Heaviside expression of the graph of figure 1.